



# How many stocks are sufficient for adequate diversification?

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• Many investors seek exposure to single stocks for various reasons.

- In this context, it is still a widely held belief that 20 to 30 stocks make for sufficient diversification.
- A growing body of literature shows that this belief roots in an overly simplistic definition of diversification and poorly designed empirical tests.
- We contribute to the field by showing the diversification benefits of holding a higher number of stocks under the assumption of tracking error optimized single stock selection.

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D espite the abundance of funds and ETFs, many end investors seek single stock exposure. This decision can be motivated by various reasons. However, regardless of whether those are rational, asset wealth managers are often forced to accommodate this preference. At the same time, especially boutique investment firms tend to be constrained in their research capacities. This problem raises the question of how an efficient portfolio can be constructed that takes clients' preferences into account without betting the bank on them.

### 1 The eternal allure of stock picking

There is a reason, why some sins have been covered equally prominently by the Old Testament and post modernist literature. Certain things in life are just too seductive and we can't resist their allure, even though we objectively know, they are bad for us. It may be the second bottle of red wine during dinner and this perfect Tiramisu served at the end. In investing, it is market timing and single stock picking. Some investors believe in their ability to spot outperforming business or think their manager of choice does (overconfidence).

Others are driven by emotional attachment to particular regions (home bias), industries (family business, profession, personal interest), or enjoy the intellectual discussion and the thrill of taking bets in the companies that shape our world. On top of that, behavioral studies have discovered that investors tend to neglect the correlation between assets resulting in naïve diversification strategies (correlation neglect hypothesis)[7]. Some of these emotional preferences can be easily satisfied if a fortune is large enough to offer a "gambling account" capacity. However, this is not always the case, and some market participants may wish to put significant emphasis on the stocks that form the centerpiece of discussions with their advisor. We also note that end investors frequently tend to focus excessively on the absolute performance of individual stocks rather than the aggregate portfolio.

This phenomenon is not surprising as it tends to be less satisfying to understand a co-variance matrix (see, for instance, Svedsater 2006 [8]) than to discuss wellknown businesses. However, it can be challenging for advisors to please their clients and comply with legal, regulatory, ethical, and professional standards. In this context, we have repeatedly faced the question of how many stocks are needed to construct an adequately diversified portfolio.

### 2 Research results have long legs

It turned out to be surprisingly challenging to find a satisfying answer. The problems start with the question, what the term diversification means and how to measure it. The Capital Asset Pricing Model, for instance, differentiates between idiosyncratic (firm-specific) and market risk. It thereby describes market risk as the portion of risk that is not diversifiable. A diversified portfolio would thus be a portfolio whose risk stems exclusively from exposure to market risk. Therefore, the question would be, how many stocks are at least needed to construct a portfolio that has negligible exposure to idiosyncratic risk. The past decades have produced, an extensive body of literature, and there are almost as many answers as researchers.

Stunningly, for instance, an article in the Journal of Finance from 1968 found that the "benefits of diversification are virtually exhausted when a portfolio contains approximately ten stocks". The American Association of Individual Investors finds that many people still believe "that diversification beyond 10 or 20 securities is superfluous" and also notes that "some investment textbooks have helped spread the confusion" [3]. One example for this kind of misleading advice is an article by Morningstar, claiming that "if you own more than 18 stocks, you will have achieved almost full diversification"[4]. Beyond that, it warns of the adverse effects of having to "keep track of more stocks in your portfolio for not much marginal benefit".

These examples showcase a general problem in Finance. Even outdated research results tend to be stubbornly sticky and can influence investor's beliefs and decisions decades later. While the ten stock ideas still ghosts around, many popular sources nowadays seem to agree with Statman (1987), who "showed that a well-diversified portfolio of randomly chosen stocks must include at least 30 stocks" [13]. Business Insider, for instance states that "the generally agreed upon number is 20 to 30 stocks." [12].

The CFA Institute just published an article in May 2021, claiming that "peak diversification is achieved with around 26 stocks" [1]. This appears to indicate a surprisingly broad-based consensus even more than 30 years after the publication of Statman's paper. The idea



Figure 1: Average portfolio standard deviation as a function of number of stocks held (random portfolios) The CFA Institute 2021

that maximum diversification is achieved with 25-30 stocks is usually supported with graphics like Figure 1. It is one of several similar charts published by the CFA Institute in 2021 that show the standard deviation of portfolios consisting of several randomly chosen stocks. It thereby follows a methodology pioneered by Evans and Archer (1968), who conducted similar experiments and concluded that 30 stocks sufficiently diversify a portfolio[6]. Indeed, the chart clearly shows that portfolio volatility decreases rapidly as the number of securities held increases until a portfolio size of 25

to 30 stocks is reached. Beyond this magical threshold, the marginal reduction in standard deviation achieved by adding further stocks seems to be minor, which is why researchers and practitioners tend to conclude, no meaningful diversification benefits can be harvested beyond it.

Unfortunately, all these studies suffer from two major shortcomings:

- They are based on the repeated construction of portfolios of randomly chosen stocks.
- They equate diversification with the reduction of portfolio standard deviation and thus ignore returns.

These limitations result in dangerous misunderstandings and confusion among investors, which is why we are separately elaborating on both problems.

# 2.1 Portfolio construction is not a Monte Carlo simulation

In their study, published by the CFA Institute, Eccles and Horstmeyer produced charts like Figure 1 by constructing "a random portfolio from a given number of equally-weighted stocks" and calculating "its volatility using monthly returns over the 15 years from 2005 to 2020". They then selected "another random portfolio of the same size" and "conducted the same procedure 100 times, averaging the volatility across all these iterations". In other words, the standard deviation shown does not reflect the volatility of any particular portfolio but the average volatility of a large number of portfolios.

Unfortunately, individual investors are not exposed to average standard deviations. Figure 1 displays the volatility faced by 100 investors on average. Still, every investor holds exactly one portfolio at a time and could hence end up with lower or higher risk. This problem was researched quite well by Raju and Agarwalla (2021)[11]. In their empirical study on the Indian stock market, they showed that a randomly compiled 20 stock portfolio diversifies away approximately 90% of diversifiable risk on average. However, using the dis-

		1yr				3yr			
Risk Reduction*	Average	99	95	90	Average	99	95	90	
$\begin{array}{ c c c c } 90.0\% \\ 95.0\% \end{array}$	20 32	78 174	$55 \\ 122$	44 98	$20 \\ 31$	76 174	55 122	43 96	

\* in diversifiable risk defined by  $\phi_4$ .

The columns reflect the average risk and the upper bounds of the 99%, 95% and 90% intervals. The rows refer to the amount of diversification risk reduced. For instance, for 90% risk reduction, we look for the number of stocks where the  $\phi_4$  is equal to 10%. The number of stock is the mean number as an integer across all the 13 periods. We are measuring diversification where n = 1,10,20...500, so the mean number is just a mathematical average.

Figure 2: Number of stocks vs volatility reduction (random portfolios)

Raju and Agarwalla (2021)

tribution of investment outcomes, they also discovered

that "in practice, investors would find that their chosen 20-stock portfolio has a low probability of realizing the expected diversification benefits". Table 2 illustrates this nicely. An investor holding a single portfolio of randomly selected stocks with a one-year time horizon already needs at least 44 securities to achieve a 90% risk reduction with a probability of 90% and at least 55 stocks to be 95% sure to realize the stipulated diversification benefit. The difference between average risk reduction and risk reduction, achieved with a certain level of confidence, becomes even more significant once the targeted risk reduction level increases to 95%. To diversify 95% of diversifiable risk with a 90% confidence, an investor would already need to hold 98 stocks. Figure 2 thus demonstrates very well where the 20 to 30 stock idea is coming from but also shows why it is problematic.

The utilization of average portfolio standard deviation is one aspect of many studies on equity portfolio diversification. Another limitation, often overlooked in this context, is the prerequisite of a completely random single stock selection. Gubaydullina et al. (2009) found that individuals "take irrelevant information as a foundation for their investment decisions," and the single stock picks of some market participants may indeed look erratic. Nevertheless, it would be a cynical exaggeration to presume that they are entirely random. In other words, a real-life 30 stock portfolio constructed even by a professional investor is very likely to inhibit significant style, country, and industry biases.

As mentioned earlier, Gubaydullina et al. (2009) also found that investors tend to neglect correlation and use naive diversification strategies. They are thus likely to pick stocks that are relatively highly correlated, resulting in portfolios that are, in fact, less diversified than the random portfolios used in the quoted studies on portfolio diversification. Beyond that, we also think that the concept of measuring diversification benefits purely based on volatility reduction is problematic because of an underlying misunderstanding of the meaning of diversification.

#### 2.2 Diversification is not just about low volatility

The approach outlined above completely ignores the role of expected returns in portfolio diversification. Elton and Gruber (1977) already addressed this short-coming by also considering the risk that a portfolio's mean return will differ from the return of the reference market portfolio (shortfall risk). Domian et al. (2007)[5] followed this approach and studied a large sample of 1000 US stocks, measuring the chance that a random portfolio will underperform US Treasuries. The study adjusts for survivorship bias and regular rebalancing while covering a relatively long period of 20 years from 1985 to 2004, including the burst of the

Dotcom Bubble.

It is noteworthy that in their sample, they find that roughly a quarter of firms generated a loss over the entire 20 year period while almost 70% delivered a total return of less than 1% per year. Bessembinder (2018)[2] confirmed these findings, studying the US market starting in the year 1926. He showed that, in the long run, only 4% of firms generated the total performance of the US equity market. Ceteris paribus, this huge skewness in the cross-section of returns implies that concentrated portfolios can yield pretty disastrous long-term results.

Not surprisingly, Domian et al. (2007) found that over the 20 years covered, even "100 stocks are not enough to provide sufficient protection from shortfall risk" (the risk of underperforming the risk-free rate). To generate returns, at least equal to the risk-free rate with 99% confidence, they required 164 stocks. Unlike many other studies, Domian et al. (2007) also studied active diversification across industries. This approach is likely more in line with the behavior of actual investors. However, according to their study, it only results in a very modest improvement of shortfall risk for highly concentrated portfolios (10 stocks) and virtually no enhancement for more extensive portfolios.

# 3 An empirical study based on nonrandom single stock selection

Using shortfall risk instead of standard deviation and considering industry diversification makes studies more realistic. However, this research design still assumes pretty unsophisticated investor behavior. What if an investor goes beyond naïve industry diversification to reduce shortfall risk? Ceteris paribus, we would expect a portfolio constructed based on some kind of advanced knowledge and insight to harvest the same diversification benefits with a lower number of stocks than the semi-random portfolios of Domian et al. (2007). To



test this assumption, we perform several tests using

the Bloomberg multi-factor risk model. First of all, we minimize total portfolio risk (standard deviation) for portfolios of various sizes (the Evans and Archer (1968) approach but with non-random single stock selection). Figure 3 shows the results for portfolios constructed with stocks that are members of the STOXX 600 Index. The experiment shows the meaninglessness of an approach purely based on portfolio volatility. We can reduce predicted portfolio volatility below the market level even with a handful of stocks. However, this results from heavily loading up on the low volatility factor and the consumer staples sector. Such an allocation is hardly adequately diversified.

We, therefore, choose to focus on active risk instead of total portfolio risk. This method is more comparable to the approach pioneered by Elton and Gruber (1977). Instead of measuring shortfall risk (the risk of ending up with wealth below a certain reference level), we thus consider the risk of involuntarily deviating from the performance of the reference market. The question our tests attempt to answer is how many stocks are at least needed to more or less eliminate undesirable active risk. It is essential to bear in mind that active risk is not unwanted per se. Investors may choose to overweight sectors, countries, styles (smart beta), or single stocks (alpha). We, therefore, perform tests from the point of view of a value-oriented investor who targets a specified exposure to this factor while minimizing residual active risk. The critical point here is that all active bets occur in a controlled way without any unintentional exposures.

#### 3.1 Covering Europe the US and Value Investing

In total, we perform four sets of tests:

- Monthly minimum active risk optimization on STOXX 600 stocks (Test 1)
- Annual minimum active risk optimization on STOXX 600 stocks (Test 2)
- Annual minimum active risk optimization on S&P 500 stocks (Test 3)
- Annual minimum active risk optimization with value bias on STOXX 600 stocks (Test 4)

Figure 4 shows some results of the second optimization that minimizes total active risk on an annual basis against the STOXX 600 Index without any constraints except for the number of stocks held. Active risk converges to 0 once we fully replicate the index with 600 stocks. However, the marginal benefit of adding more stocks across all samples becomes pretty tiny once we hold more than 200 stocks. We backtest this active risk optimization over the past ten years for portfolios holding 20, 30, 50, 80, 90, 100, and 150 stocks. It allows us to monitor how the active risk of different portfolios evolves and how their performance compares to that



of the market. We can also compare the expected (exante) risk given by the optimizer with realized (ex-post) risk provided by actual performance. In this context, we define ex-post active risk as the standard deviation of the delta between daily market and daily portfolio returns. Finally, we present all statistics on an annualized basis. Figure 19 illustrates this for the European equity universe with annual portfolio formation. Results for all other tests are given in Figure 18 to Figure 21 in the appendix. We can see that there is virtually



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no difference in the volatility of the six portfolios. This is in line with earlier findings that a low standard deviation of returns can be obtained with a pretty low number of stocks. In addition, however, we observe a significant and consistent reduction in active risk the more stocks we add. It is worth noting that for systematic long-only equity strategies, institutional investors usually demand a tracking error below 3%[10]. The 20 stocks portfolio, despite minimizing tracking error with virtually no other constraints, clearly fails this requirement. While the 30 stocks portfolio reports average tracking error below this threshold, the three months average active risk frequently exceeds it during times of financial distress. To consistently keep active risk below 3% with annual portfolio formation, we need to hold at least 80 stocks. While even 4% (annualized) tracking error may sound like a low number

to many investors, it actually has a significant impact on portfolio performance over the analyzed ten-year time horizon. This becomes clear once we look into



Figure 6: Cumulative performance difference (European universe, monthly portfolio formation) Amadeus Capital SA

the cumulative performance difference between each of the six portfolios and the market index as in Figure 14 to Figure 17. When we construct the portfolios on a monthly basis, only the 100 stocks portfolio performed in line with the market (before cost) over this time. The



Figure 7: Cumulative performance difference (European universe, annual portfolio formation) Amadeus Capital SA

European 20 stocks performance, for instance, with its 3.5% annual tracking error, underperformed the market by a whooping 38%, and even the 80 stocks portfolio was still 17% worse than its benchmark. With annual portfolio formation, the 100 stocks portfolio drifts away from the market as well, and we need at least 150 stocks to control tracking error across the ten-year investment horizon effectively.

We repeat the same exercise for a US universe consisting of all S&P 500 members. Here, we observe a strong outperformance of the 20 and 30 stock portfolios as shown in Figure 16. On the other hand, the 50 stock portfolio returned less than the market. In this context, the focus shouldn't be on the direction of relative performance but the illustration of the vast deviation of the more concentrated portfolio from the market. As we construct all portfolios intending to minimize active risk, any deviation from the benchmark is, by



Figure 8: Cumulative performance difference (US universe) Amadeus Capital SA

definition, undesirable risk.

#### 3.2 Holding more stocks may be cheaper

We also study the relationship between the number of stocks held and portfolio turnover and find a pronounced negative correlation between the number held and portfolio turnover even when we invest in more than 100 stocks. Figure 27 shows this for the European portfolio constructed on an annual basis. Again, further illustrations are given in the appendix in Charts 26 to 29. This relationship indicates that investors can sub-



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stantially reduce transaction costs by holding a higher number of stocks. The effect is especially relevant for large investors where market impact is a concern. (Of course, this is not the case for retail investors with small portfolios who pay a minimum commission for each security traded).

#### 3.3 Extending the framework to active strategies

Finally, we run the same minimum active risk optimization but with an active overweight on the value factor. Figure 17 shows the results obtained for the STOXX 600 universe. In this case, the underperformance of the 20 and 30 stock portfolios is even more pronounced than in our first example. This finding is not too surprising as we would expect the factor bias further to reduce the already poorer diversification of the concentrated portfolios. Figure 17 compares the tracking



Figure 10: Cumulative performance difference (European universe, annual portfolio formation, value bias) Amadeus Capital SA

error of the value portfolio with the unconstrained portfolio. We note that beyond a portfolio size of 90 to 100 stocks, the marginal benefits of adding additional securities seem to be smaller for the value portfolio than for the unconstrained portfolio. For our value portfolio, the lowest tracking error is achieved with an average portfolio size of 306 stocks but annualized active risk drops by only 0.1% when we increase the number of stocks from 100 to 306. Around 100 stocks, we reach a threshold where it becomes difficult to reduce tracking error further while maintaining the desired active style bias. However, it is worth noting that we still achieve a roughly 14% reduction in annual portfolio turnover. Of course, an annualized turnover of 60%-70% is pretty high for a tracking error-controlled value strategy. In practice, we would control for turnover by adding respective constraints.



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#### 3.4 400 stocks may be an overkill

We also simulate a portfolio with 150 stocks and a slightly more pronounced value bias and compare

it with the Amundi MSCI Europe Value ETF. The fund holds around 400 securities and assigns security weights semi-annually based on stock's value score and market capitalization[9]. Figure 12 shows the per-



formance (adjusted for cost) in comparison to our 150 stocks portfolio as well as the EURO STOXX 600 Index. We note that the performance of our value portfolio is pretty close to that of the MSCI Index, indicating that it has similar exposure to the value factor. However, our



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tracking error-optimized solution comes with roughly 1% lower volatility and 0.6% lower active risk.

# 4 Conclusion - don't skip the free lunch

A sizable body of literature covers the topic of single stock diversification. Unfortunately, many well-known studies approach the problem in an overly simplistic way. The idea that diversification benefits are minimal beyond a portfolio size of 20-30 stocks stems from the methodology pioneered by Evans and Archer (1968). This approach calculates the average standard deviation of randomly drawn portfolios containing a certain number of stocks. Later publications such as Domian et al. (2007) [5] already pointed out that this approach suffers from severe shortcomings.

Most importantly, it completely ignores returns and the fact that real-world investors only buy one portfolio and not the average of a range of simulated portfolios. Domian et al. (2007) showed that investors could significantly reduce return dispersion between randomly drawn portfolios and expected shortfall compared to a minimum return threshold by adding more than 30 stocks. They suggest that investors need no less than 174 stocks to remove 95% of diversifiable risk with 90% confidence.

We go beyond this framework by testing the risk and performance of optimized portfolios utilizing the Bloomberg multi-factor risk model. In this context, we use active risk compared to the market portfolio (in this case, the STOXX 600 Index and the S&P 500 Index) as the primary measure of risk, which we minimize. We show that concentrated holdings of 20, 30, and even more than 50 stocks can result in substantial performance deviations, even if the market participants try to stay as close to the market as possible. The results are even more extreme when we form portfolios that actively target the value factor. Furthermore, we find a robust negative correlation between the number of stocks and portfolio turnover, implying that investors can meaningfully reduce transaction and active risk at the same time by increasing the number of stocks held.

We find that European and US investors need roughly 150 securities to reliably replicate the performance of the reference market if they construct and rebalance their portfolios on an annual basis. Backtests indicate that a strategy based on 100 stocks may be sufficient with more frequent updates but likely results in much higher turnover. In our tests, an optimized strategy that minimizes active risk while maintaining a specified exposure to the value factor outperformed the MSCI Value Index on a risk-adjusted basis. Our portfolio thereby holds only 150 stocks instead of the 400 stocks held by the index indicating that portfolio optimization trumps a higher number of stocks.

Factor or risk premia investing is only one possible application of our optimization-based approach.

Resource-constrained investors with strong views on a limited number of securities could also use a similar strategy to build around their core picks and thus reduce risk and improve diversification without losing the opportunity to generate alpha through informed single stock picks. In either case, the still widely held belief that 20 to 30 stocks make for good diversification results from a dangerous misunderstanding.

Of course, there is nothing wrong with a sophisticated investor making a bold call on a handful of stocks. But market participants who follow such an approach should be aware of the facts that

- they are deliberately giving up diversification benefits in return for the opportunity to earn higher returns on their alleged skill;
- the odds are clearly against them (see, for instance, Bessembinder (2018)[2]);
- they are likely suffering from overconfidence and other behavioral biases.



Figure 14: Performance comparison Test 1



Figure 15: Performance comparison Test 2



Figure 16: Performance comparison Test 3



Figure 17: Performance comparison Test 4





Figure 21: Active risk/volatility comparison Test 4



Figure 22: Number of stocks vs active risk comparison Test 1



Figure 23: Number of stocks vs active risk comparison Test 2



Figure 24: Number of stocks vs active risk comparison Test 3



Figure 25: Number of stocks vs active risk comparison Test 4



Figure 26: Number of stocks vs turnover comparison Test 1



Figure 27: Number of stocks vs turnover comparison Test 2



Figure 28: Number of stocks vs turnover comparison Test 3



Figure 29: Number of stocks vs turnover comparison Test 4

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